Mathematics Challenge 2015

by

Children's Well-wishers Network (CWN)

YEAR 6

Mark Scheme

We provide mark schemes of our CWN Mathematics Challenge 2015 examination papers to help parents.

Please note that for some problems there are more than one possible answer.

Some questions are open ended.

We strongly advise all children to practise the papers and think hard before looking at the answers provided.

Full answers and explanations will be provided on our feedback sessions.

In general, we expect units, directions, sensible answers and reasons in all questions.

Q1) If **a** is equal to **b²** and b is a positive whole number then we call **a** as the square number of **b** and at the same time **b** as the square-root of **a.**

We can write this as: $b = \sqrt{a}$

(a) What is the square-root of 196?

14

(½ mark)

(b) What is $\sqrt{2}$ to one decimal place?

1.4 (to 1 d.p.)

(½ mark)

(c) There are many ways to workout $\sqrt{2}$.

Below is a method called continued fractions. The ellipsis (…) means the representation continues indefinitely.

$$\sqrt{2} = 1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \ldots}}}}$$

Using the truncated continued fraction representation of $\sqrt{2}$ below:

$$1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2}}}}$$

work out the approximated fraction value for $\sqrt{2}$.

$1\frac{12}{29}$

(1 mark)

Q2)

(a) Ahmed bought $5\frac{3}{4}$ cans of white paint. He used $2\frac{2}{3}$ cans of it while painting a room. How much of white paint was he left with?

Answer: $3\frac{1}{12}$ cans

(1 mark)

(b) He then mixed the left over white paint with new blue paint in the ratio 1:2 respectively. How much of paint did he have afterwards?

$9\frac{1}{4}$ cans

(1 mark)

Q3)

(a) How many different digits will appear when $\frac{34}{33}$ is written as a recurring decimal?

Three

(1 mark)

(b) Express 0.375 as fraction in the simplest (reduced) fraction form.

$\frac{3}{8}$

(1 mark)

Q4) Write a number in the box to make the following true:

Many possible answers

(a) 6.3 > any number that is less than 6.3 (not 6.3)

(½ mark)

(b) any number that is less than 6.3 (not 6.3) < 6.3

(½ mark)

(c) 6.3 < any number strictly in between 6.3 and 6.4 < 6.4

(½ mark)

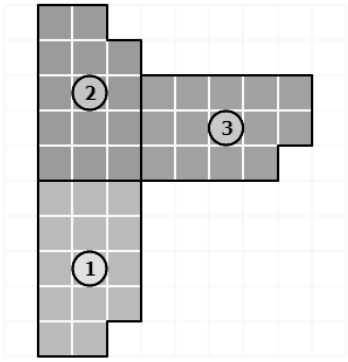
(d) Henry's school bag weighed 6.7 kg. He took out a textbook weighing 0.87 kg and her lunch box weighing 0.505 kg. What does her bag weigh now?

5.325 kg

(½ mark)

Q5)

(a) The shape ① is transformed to ② and then to ③ as shown below.



Describe the transformation.

Tick (✓) the correct answer.

- [ ] Rotation, then translation
- [✓] Reflection, then rotation
- [ ] Reflection, then translation
- [✓] Rotation, then reflection

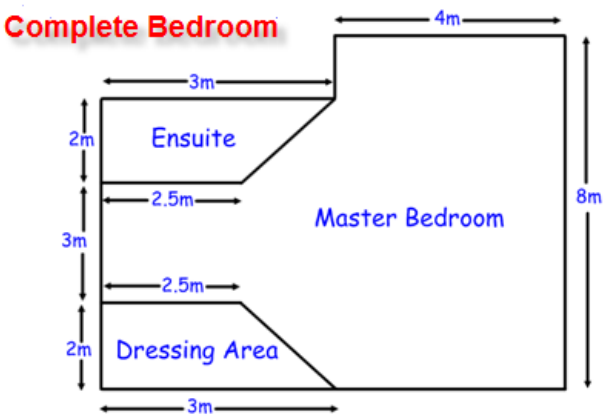
(1 mark)

(b) Describe the inverse transformation that will bring the shape back to the original position?

Rotation (anticlockwise), then reflection (on the horizontal bottom edge).

(1 mark)

Q6) Look at the bedroom plan and answer the questions below:



Drawn not to scale

(a) What is the perimeter of the complete bedroom?

30 m

(1 mark)

(b) What is the perimeter of the master bedroom?

$(8 + 4 + 1 + \frac{\sqrt{17}}{2} + 2.5 + 3 + 2.5 + \frac{\sqrt{17}}{2} + 4)$ m

$= (25 + \sqrt{17})$ m

or 29 m roughly

(1 mark)

(c) What is the area of the complete bedroom?

53 m^2

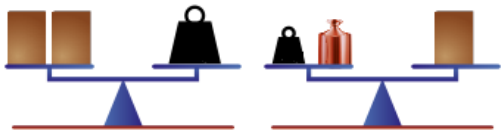
(1 mark)

(d) What is the area of the ensuite?

5.5 m^2

(1 mark)

Q7) Here are two *scales*. Both scales are balanced.



There are two **known** black weights and one **unknown** copper weight.
The big black weight on the left scale is **B** kg.
The small black weight on the right scale is **s** kg.
The unknown copper weight on the right scale is **U** kg.

Brown boxes have same weight.

Write a relationship between **U**, **B** and **s**.

B = 2s + 2U

(1 mark)

Q8)

(a) Write the **largest** whole number for **n** to make this statement true: $n^2 < 50$

7

(½ mark)

(b) Write the **smallest** whole number for **n** to make this statement true: $n^2 > 50$

Negative infinity
or indetermined
or undefined

(½ mark)

(c) Write the **smallest** whole number for **n** to make this statement true:

$$n^2 < 2n$$

1

(½ mark)

Q9) A sequence starts at 32 and is halved each time to get a new term.

32 16 8 ...

The sequence continues in the same way.

(a) Write the first two numbers in the sequence which are less than one.

½ and ¼

(½ mark)

(b) After how many terms the sequence will generate negative terms?

It will never generate negative terms.

(½ mark)

Q10)

(a) A fence post in a garden is 4 feet tall. On a late evening it casted a shadow of 12 feet long. At the same time an apple tree very close to the fence post casted a shadow of 72 feet.
How tall was the tree?

24 feet

(½ mark)

(b) Two squares **A** and **B** have lengths in the ratio **2 : 3** .

i. What is the ratio between their perimeters in that order?

2 : 3

(½ mark)

ii. What is the ratio between their areas in that order?

4 : 9

(½ mark)

Q11)

(a) Identify the continuous data in the following list.

Tick the correct answer(s).

- [✓] Time taken to have dinner
- [✓] Height of a tree

(Both should be ticked to award 1 mark)

- [ ] Shoe sizes
- [ ] Dates on a calendar

(1 mark)

(b) Identify the discrete data in the following list.

Tick the correct answer(s).

- [✓] Number of students in your class
- [ ] Weight of cars
- [ ] Speed of a train
- [ ] Length of a pencil

(1 mark)

Q12) What pattern do you notice below?

$$76{,}923 \times 1 = 076{,}923$$
$$76{,}923 \times 10 = 769{,}230$$
$$76{,}923 \times 9 = 692{,}307$$
$$76{,}923 \times 12 = 923{,}076$$
$$76{,}923 \times 3 = 230{,}769$$
$$76{,}923 \times 4 = 307{,}692$$

Award ½ mark for:
- Same digits appear in all products.

Award 1 mark for:
- Products are written in the cyclic order of the same digits (or any valid answer that contains mentions the digits being used are the same and the order in which they are displayed)

(1 mark)

Q13)

(a) You invite 3 friends for a dinner. You have a round table and you sit in your preferred chair. You ask your friends to sits in the other chairs.

In how many different ways can they be seated?

6

(1 mark)

(b) Ronnie is ordering a birthday cake for a friend.

The bakery makes chocolate and lemon flavoured cakes. Each flavour of cake can come with green or pink frosting. He can also pick peach, fudge, coconut, or almond filling.

How many different combinations can Ronnie choose from?

2 x 2 x 4

16

(1 mark)

(c) A boy and a girl are talking.

"I am a boy" - said the child with brown hair.
"I am a girl" - said the child with black hair.

At least one of them lied. Who lied?

Both

(1 mark)

Q14) A bag contains 3 red and 5 green marbles. If you took out a marble with your eyes closed, what chance would you give to each of these outcomes?

Express the likelihood of each of the events **(a)** to **(f)**.

(a) The marble taken out is red.

$\frac{3}{8}$

(½ mark)

(b) The marble taken out is red and green.

0

(½ mark)

(c) The marble taken out is not green.

$\frac{3}{8}$

(½ mark)

(d) The marble taken out is red or green.

1

(½ mark)

(e) If you take out a marble, put it back again, then take out a second marble, both marbles will be red.

$\frac{9}{64}$

(1 mark)

(f) If you take out a marble and then take out a second marble, both marbles will be green.

$\frac{5}{14}$

(1 mark)

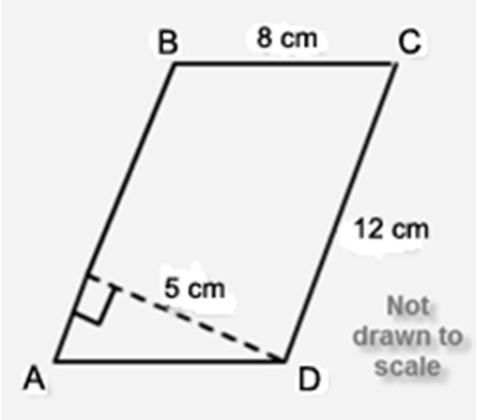
Q15) A sink contains exactly 20 litres of water. If water is drained from the sink until it holds exactly 6 litres of water less than the quantity drained away, how many litres of water were drained away?

13 (litres)

(1 mark)

Q16)

(a)

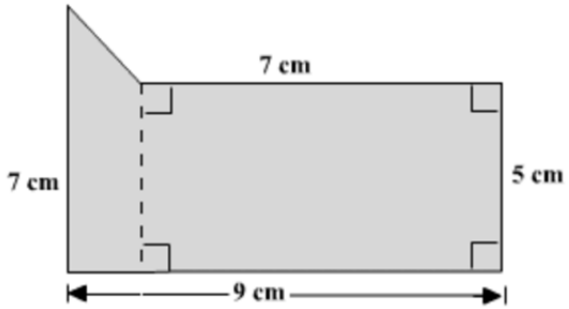


What is the distance between BC and AD of the parallelogram?

7.5 cm

(1 mark)

(b) Find the area of the compound shape below using the dimensions given.



Area: 47 cm^2

(1 mark)

Q17) These pie charts show the flavours of ice creams sold in two shops: Shop A, Shop B.

(a) In which shop were more chocolate ice creams sold?

We cannot tell.

(1 mark)

(b) If Shop B sold 720 ice creams how many vanilla ice creams were sold in the same shop?

180

(1 mark)

Q18) In Statistics, the **range** of a set of numbers is the difference between highest and lowest values of that set.

Write a number in each of these boxes so that the **mean** of the five numbers is 11 and the **range** is 4.

Couple of possible answers:

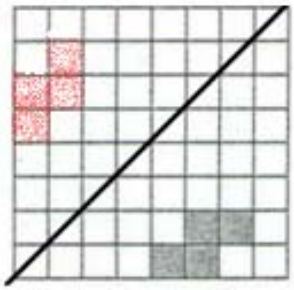
9 11 11 11 13

8 11.5 11.5 12 12

(1 mark)

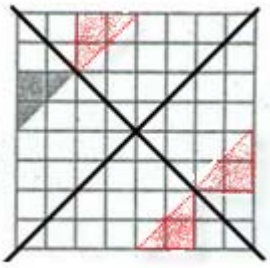
Q19)

(a) Shade in squares so that the shapes are symmetrical about the black line of symmetry.



(1 mark)

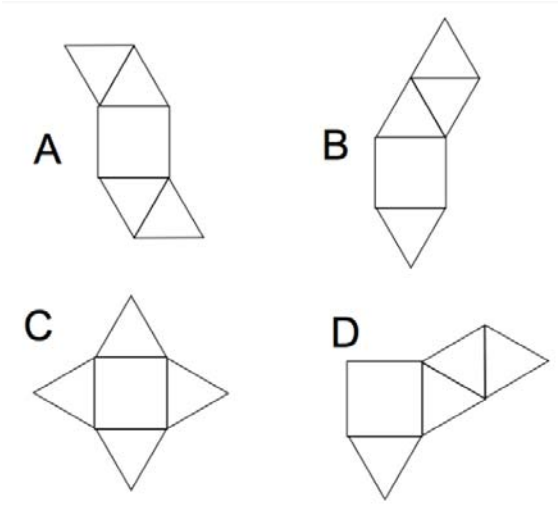
(b) Shade in squares so that the diagrams are symmetrical about the two black lines of symmetry.



(1 mark)

Q20) Here are 4 diagrams.

One of them is not the NET of a famous solid.



(a) What is the name of the solid mentioned above?

Square-based pyramid

(1 mark)

(b) Which diagram is not a NET of the solid? Write the letter.

B

(1 mark)

Q21) Find two pairs of integers that satisfy the equation:

$3x + 4y = 25$

(a) x = 3

y = 4

(1 mark)

(b) x = 7

y = 1

(1 mark)

Q22)

(a) In a sale, the prices of all items in a shop were decreased by 20%.
After the sale they were all increased by 25% from the reduced price.
What was the overall effect on the shop prices?

Nil

(½ mark)

(b) Explain how you know.

((original price x 0.8) x 1.25 = original price)

(½ mark)

Q23) In the Indian epic Mahabharatam the villain Shakuni scored 6 consecutively four times when he rolled a normal six sided die.

The record of results:

Trial	1st roll	2nd roll	3rd roll	4th roll	5th roll
Score	6	6	6	6	

Lord Krishna looked at the results and disputed with Shakuni claiming:

Your die was not a fair die.

Was Lord Krishna correct?

Circle Yes or No: **Yes** / **No**

(½ mark)

Give a reason for your answer.

It is unlikely (= 1 in 6^4 chance) to get a 6 four times consecutively rolling fair die.

(½ mark)

Q24) The King asks Archimedes if his crown is made from pure gold.

He knows that the crown is either pure gold or it may have some silver in it.

Archimedes figures out that the volume of the crown is 125 cm^3 and that its mass is 1.8 kilograms.

He also knows that :

- 1 kilogram of gold has a volume of about 50 cm^3.
- 1 kilogram of silver has a volume of about 100 cm^3.

(a) Is the crown pure gold?

No

(½ mark)

(b) Explain how you know.

If it was made of pure gold the volume of gold should have been 90 cm^3 rather than 125 cm^3

(½ mark)

(c) How many kilograms of gold and silver does the crown contain?

Density of gold = 0.02 kg/cm^3
Density of silver = 0.01 kg/cm^3
Let gold in the crown be x kg
Let silver in the crown be y kg

$x + y = 1.8$
$x / 0.02 + y / 0.01 = 125$

so

gold in the crown 1.1 kg
silver in the crown 0.7 kg

(2 marks)

Q25)

(a) A perfect number is the one having the property that the sum of its factors (excluding the number itself) equals the number.

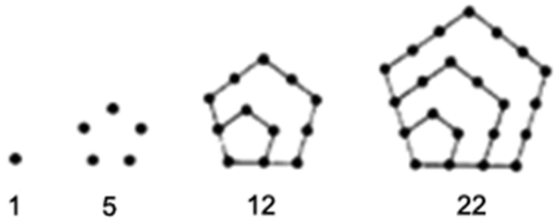
Prove 28 is a perfect number.

Factors of 28 excluding 28 are 1, 2, 4, 7, 14.

1 +2 + 4 + 7 + 14 =28

(1 mark)

(b) The first four Pentagonal numbers are:



What are the next two pentagonal numbers?

Generating Sequence: n (3n – 1) /2
35 and 51

(1 mark)

(c) Write down the next two terms in each of these sequences:

i. 18, 46, 94, 63, 52, 61, 90, 40, 10

(1 mark)

ii. 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89

(1 mark)

