



Mathematics Challenge 2015

by

Children's Well-wishers Network (CWN)

YEAR 8

Mark Scheme

We provide mark schemes of our CWN Mathematics Challenge 2015 examination papers to help parents.

Please note that for some problems there are more than one possible answer.

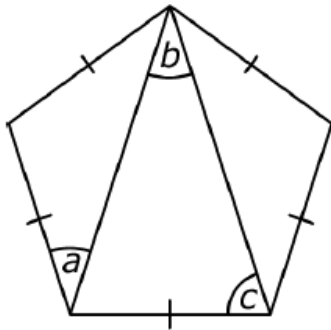
Some questions are open ended.

We strongly advise all children to practise the papers and think hard before looking at the answers provided.

Full answers and explanations will be provided on our feedback sessions.

In general, we expect units, directions, sensible answers and reasons in all questions.

Q1) A regular polygon is shown below. Work out the unknown angles a , b and c .



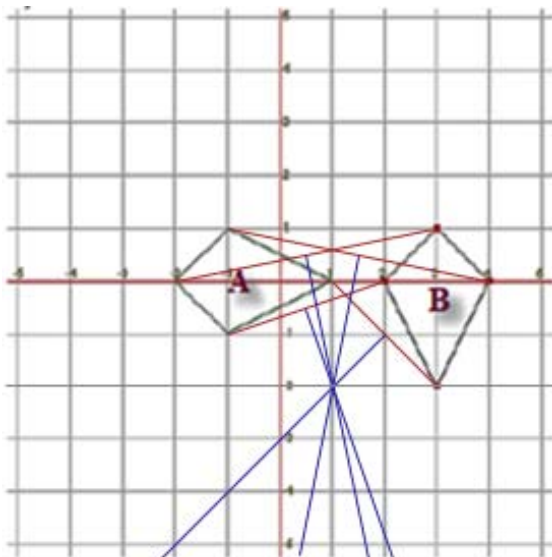
$a = 36^\circ$ (1 mark)

$b = 36^\circ$ (1 mark)

$c = 72^\circ$ (1 mark)

Q2)

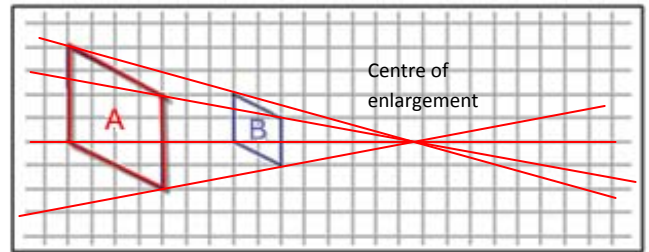
(a) Find the centre of this 90 degree rotation:



Centre of rotation: $(1, -2)$

(1 mark)

(b) Show the centre of enlargement from A to B in the grid.



(1 mark)

(c) In the above transformation the linear scale factor is:

$$\frac{1}{2}$$

(1 mark)

Q3)

(a) CWN News headline was:

"Interest Rates Jump From 10% to 12%"

This means the rise is 2 percentage points. What was the real percentage increase?

$12/10 = 1.2 = 120\%$, so that is a 20% rise.

($\frac{1}{2}$ mark)

(b) A store purchased a fountain for £100 and marked it up by 200%. Joseph makes a 12% commission on all of his sales. How much commission did he make when he sold the fountain?

£36

($\frac{1}{2}$ mark)

Q4) A normal six-sided dice is thrown 600 times. It lands 150 times on 4.

(a) What is the relative frequency of the dice landing on 4?

$$\frac{1}{4}$$

(½ mark)

(b) Is this dice biased? Explain your answer.

Yes, because the experimental probability ($\frac{1}{4}$) is not close to the theoretical probability ($\frac{1}{6}$).

(1 mark)

Q5) The marked price of a computer set is £1500. It can be purchased using either one of the schemes below:

- **Cash Scheme:** 15% Discount
- **Hire Purchase Scheme:** Down payment: 10 % and then Monthly £100 only for 24 months

Find the difference between the Hire Purchase price and the cash price.

Cash Price = £1275

Hire Price = £2550

Difference = £1275

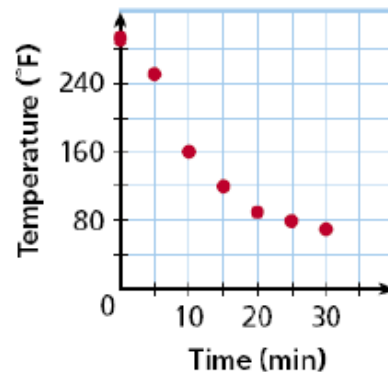
Answer: £1275

(1 mark)

Q6) There are 3 scatter plots below that represent the relationship between the number of minutes since a pie has been taken out of the oven and the temperature of the pie.

Explain each relationship indicating sensibility:

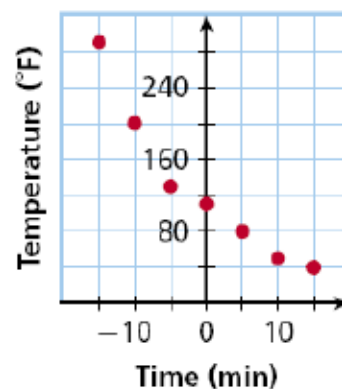
(a)



- Temperature decreases with time exponentially.
- Accept “negative correlation”
- This is sensible.

(1 mark)

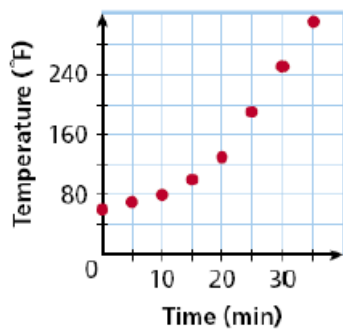
(b)



Temperature decreases with time exponentially however, as negative time has no meaning, this graph is senseless.

(1 mark)

(c)



Temperature after the pie was taken out the oven increases with time. This is senseless.

(1 mark)

Q7) Below is Kamala’s work solving an equation.

Original equation:

$$2x + 2(x - 1) = 4(x - 3) + 10$$

Expand:

$$2x + 2x - 2 = 4x - 12 + 10$$

Combine like terms:

$$4x - 2 = 4x - 2$$

Add 2 to both sides:

$$4x = 4x$$

Subtract 4x from both sides:

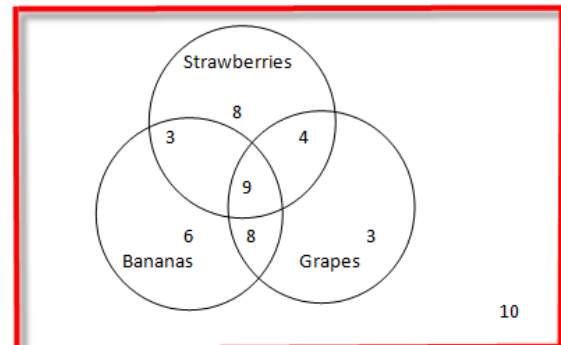
$$0 = 0$$

Kamala says: “There is no solution.”
Is she correct? Explain.

She is not correct.
The original equation is an identity. It is true for all values of x.

(1 mark)

Q8) The Venn diagram below shows the favourite fruits of a Year 8 class.



i. How many children didn’t like any of these fruits?

10 (½ mark)

ii. How many children liked only bananas?

6 (½ mark)

iii. How many children liked bananas and strawberries?

12 (½ mark)

iv. How many children liked strawberries and grapes?

13 (½ mark)

v. How many children liked only bananas and grapes?

8 (½ mark)

vi. How many children liked mangoes?

We cannot tell. (½ mark)

Q9)

(a) Solve fully for t :

$$3 - 2t > 8$$

$$-2t > 5$$

$$t < -\frac{5}{2}$$

Or $t < -2\frac{1}{2}$

(1 mark)

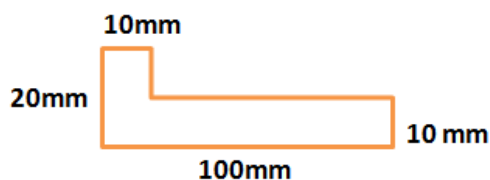
(b) Solve for the real number r :

$$4r^2 = -9$$

There are no real numbers that satisfy this equation.

(1 mark)

Q10)



The diagram shows the plan of a field.
The scale ratio is 1 : 1000
All angles are right angles.
What is the area of the field in hectares?

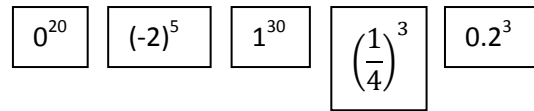
1 hectare = 10000 sq. metre.
1 sq metre = 1000000 sq. mm
Thus 1 hectare = 10^{10} sq. mm
Area in sq. mm = 1100×1000^2 sq. mm

Thus area in hectare
= $1100 \times 1000^2 \div 10^{10}$ hectares.
= 1.1×10^{-1} hectares

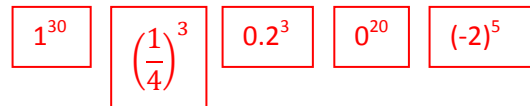
(2 marks)

Q11)

(a) Write these numbers in **descending** order:

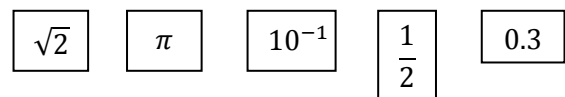


Answer:

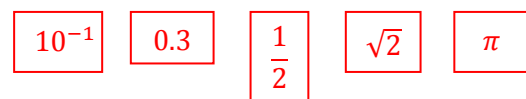


(1 mark)

(b) Write these numbers in **ascending** order:

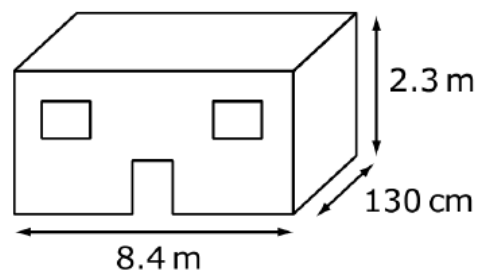


Answer:



(1 mark)

Q12) Find the volume of hut below if the model was built with the linear scale factor 1: 10.



Volume in cm^3 : $25,116,000,000 \text{ cm}^3$

Volume in m^3 : 25116 m^3

(2 marks)

Q13) The scores for three consecutive tests undertaken by a Year 8 group are presented in the table below:

	Test 1 (score out of 20)	Test 2 (score out of 50)	Test 3 (score out of 100)
Lowest score	5	12	20
Highest score	19	43	75
Lower quartile	9	19	35
Upper quartile	15	35	70

Which of the following statements are true?

1. The highest and lowest scores for the year group declined over the three tests.
2. There was no change in the year group's performance over the three tests.
3. The scores for the year group improved over the three tests.

Answer: **1**

(½ mark)

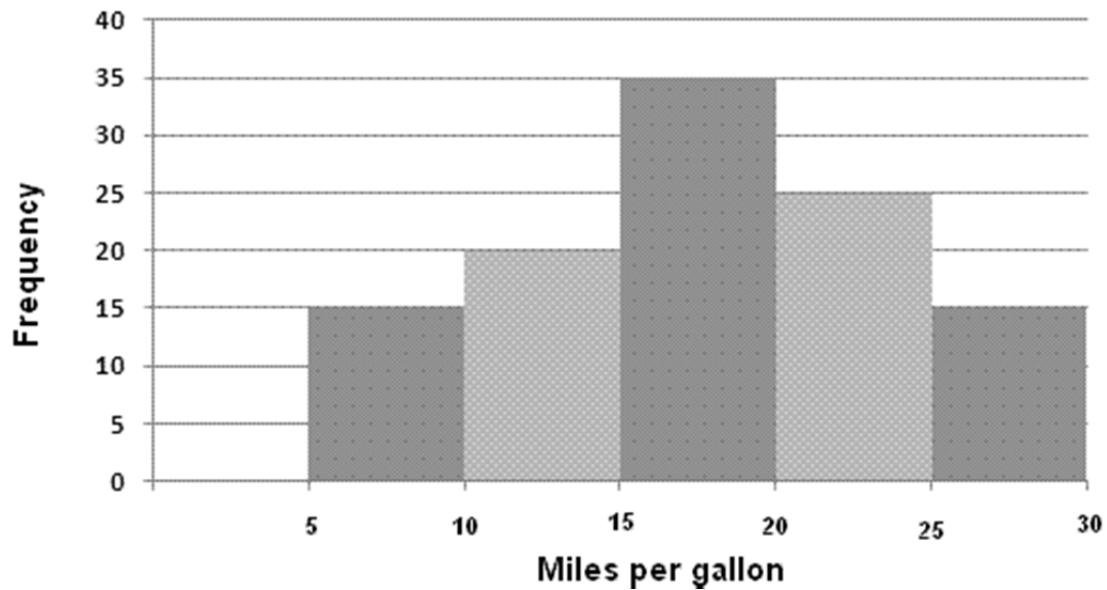
Give your reason(s).

Whilst the number of marks awarded in each test increase over the three tests, the scores (as a percentage) decrease.

	Test 1 (score out of 20)	Test 2 (score out of 50)	Test 3 (score out of 100)
Lowest score	5 = 25%	12 = 24%	20 = 20%
Highest score	19 = 95%	43 = 86%	75 = 75%
Lower quartile	9	19	35
Upper quartile	15	35	70

(½ mark)

Q14)



The histogram above shows the efficiency level (in miles per gallons) of 110 cars.

(a) What is the mean efficiency to the nearest miles per gallon?

$$\begin{aligned}
 \text{Total} &= 7.5 \times 15 + 12.5 \times 20 + 17.5 \times 35 + 22.5 \times 25 + 27.5 \times 15 \\
 &= 2.5 \times 5 (3 \times 3 + 5 \times 4 + 7 \times 7 + 9 \times 5 + 11 \times 3) \\
 &= 12.5 (9 + 20 + 49 + 45 + 33) \\
 &= 12.5 \times 156 \\
 &= 50 \times 39
 \end{aligned}$$

$$\begin{aligned}
 \text{Mean} &= 50 \times 39 / 110 \\
 &= 195/11 \\
 &= 18 \text{ miles/gallon}
 \end{aligned}$$

(1 mark)

(b) How many cars have an efficiency of more than 20 miles per gallon?

40
(½ mark)

(c) How many cars have an efficiency of 4 miles per gallon?

0
(½ mark)

(d) How many cars have an efficiency of 7 miles per gallon?

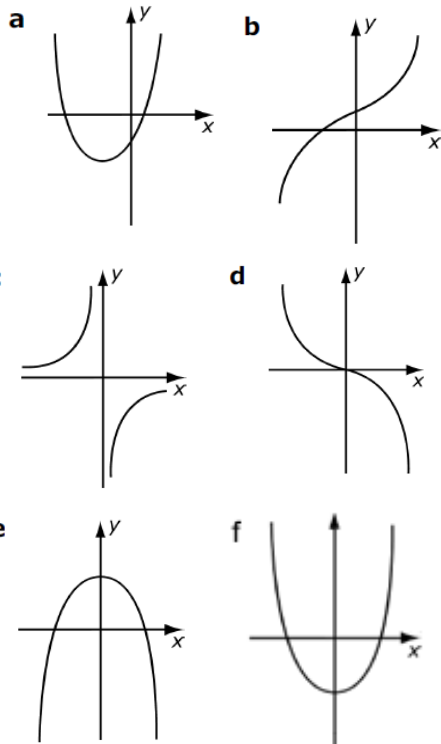
We cannot tell.
(½ mark)

(e) What percentage of cars has efficiency less than 20 miles per gallon?

$$\begin{aligned}
 &(70/110) * 100\% \\
 &= 63.63\% \quad \text{or} \quad 63\frac{7}{11}\% \quad \text{or} \quad 64\%
 \end{aligned}$$

(½ mark)

Q15) Match the equations with the graphs by writing the appropriate letter next to it.



$y = x^3 + 4$ **b**

$y = x^2 - 5$ **f**

$y = \frac{-1}{x}$ **c**

$y = -x^3$ **d**

$y = x^2 + 2x - 3$ **a**

$y = 5 - x^2$ **e**

One or two correct answers = ½ mark
 Three correct answers = 1 mark
 Four or five correct answers = 1½ marks
 All six correct answers = 2 marks
(2 marks)

Q16) Add 7×10^{-70} and 4×10^{-69} .

Write the answer in standard form.

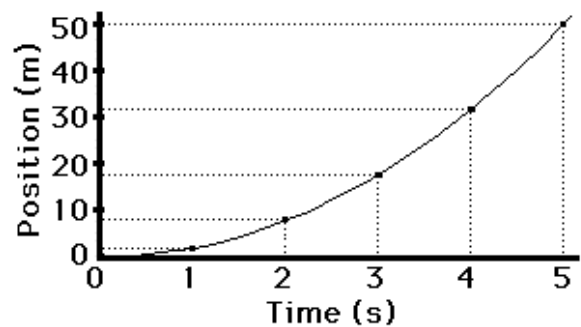
$$7 \times 10^{-70} + 4 \times 10^{-69}$$

$$= (7 \times 10^{-1} + 4) \times 10^{-69}$$

$$= 4.7 \times 10^{-69}$$

(1 mark)

Q17)



(a) In the above graph how does the position vary with time?

Quadratically (or $p = 2t^2$)

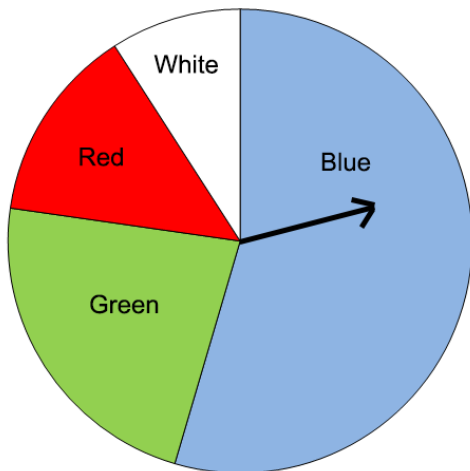
(1 mark)

(b) Work out the average velocity between the 4th and 5th second.

18 m/s

(1 mark)

Q18) You spun this biased spinner 100 times.



(a) Record a possible result of your experiment in the table below.

Any sensible values recorded below that add upto 100
and $\text{Blue} > \text{Green} > \text{Red} > \text{White}$

Color	Frequency
Blue	
Green	
Red	
White	

(1 mark)

(b) As per your experiment, what is the probability of the spinner landing on Red?

Give your answer as a fraction.

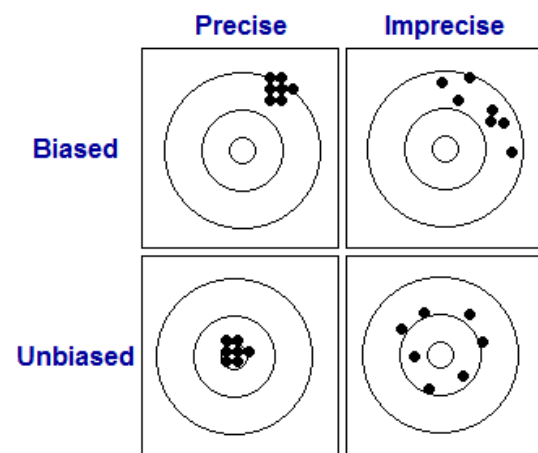
As per the above trial / 100

(1 mark)

Q19) **Bias** is a term which refers to *how far* the average statistic lies from the parameter it is estimating, that is, the error which arises when estimating a quantity.

Precision is a measure of *how close* an estimator is expected to be to the true value of a parameter.

The following illustrates bias and precision, where the target value is the bullseye:



The police decided to estimate the average speed of drivers using the fast lane of the motorway and considered how it could be done.

(a) One method suggested was to tail cars using police patrol cars and record their speeds as being the same as that of the police car. This was likely to produce a biased result. Why?

When the driver see that the police van is patrolling he/she will tend to control the speed thus the result will be biased.

(1 mark)

(b) The police then decided to use an unmarked car for their investigation using a speed gun operated by a constable. This was an unbiased method of measuring speed, but was imprecise compared to the above method. Why?

The speed gun operated by a constable will give variable results depending on his/her dexterity and response time.

Hence this method is not precise.

(1 mark)

Q20)

(a) Peter deposits £1000 in a savings account. The savings account accrues **compound** interest at a flat rate of 10% per annum.

If the interest is calculated at end of year, how much will the account be worth in 2 years?

£ 1210

(1 mark)

(b) He withdraws £100 at the beginning of second year how much will be in the account after 2 years?

£ 1100

(1 mark)

(c) John buys £2500 of shares.

He receives an annual dividend of 5% of his investment as cash into his current account.

He wants to go on a holiday using only the cash that he has received in this manner.

He estimates the holiday will cost £350.

Assuming that he does not receive interest on his current account, after how long will he be able to afford his holiday?

3 years

(1 mark)

Q21) x and y are two different integers taken from 1 to 100.

(a) What is the largest value that $\frac{x+y}{x-y}$ can have?

199

(1 mark)

(b) What is the largest value that $\frac{x-y}{x+y}$ can have?

$\frac{99}{101}$

(1 mark)

Q22) In this question x, y, z are lengths.

Circle the expressions that could represent area among the following formulae.

$$\pi x \sqrt{y^2 + z^2}$$

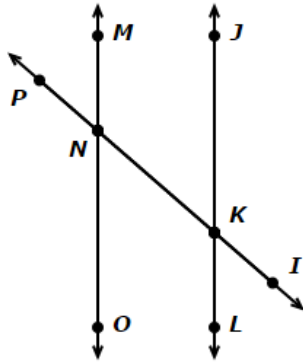
$$\pi(x+y)z$$

$$\frac{5(x+y)}{z}$$

$$\frac{3xyz}{(x+y+z)}$$

(1 mark)

Q23) \vec{JL} and \vec{MO} are parallel lines.



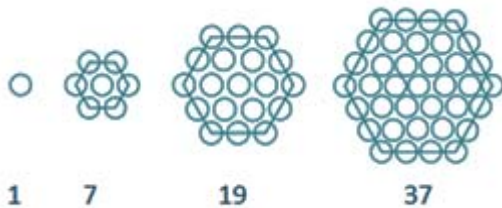
Which are adjacent angles?
Tick the correct answer(s).

- $\angle MNK$ and $\angle ONK$
- $\angle MNK$ and $\angle JKI$
- $\angle MNK$ and $\angle ONP$
- $\angle MNK$ and $\angle LKN$

(1 mark)

Q24)

(a) Hexagonal-centred numbers are found as follows.

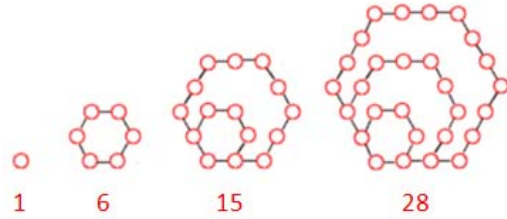


Write the next three hexagonal-centred numbers.

61, 91, 127

(1 mark)

(b) Hexagonal numbers are found as follows.



Write the next two hexagonal numbers.

45, 66

(1 mark)

(c) Find the first three terms of the sequence defined below, where n represents the position of a term in the sequence. Start with $n = 1$.

$$2(3)^n - n$$

5, 16, 51

(1 mark)

Q25)

(a) Calculate:

$$((1-1) - 1 \times 2) - (1 - (1-1) \times 2)$$

$$\begin{aligned} &= ((1-1) - 1 \times 2) - (1 - (1-1) \times 2) \\ &= (0 - 1 \times 2) - (1 - 0 \times 2) \\ &= (0 - 2) - (1 - 0) \\ &= (-2) - (1) \\ &= -3 \end{aligned}$$

(½ mark)

(b) Work out : $4 \times 6 + 18 \div 2$

33

($\frac{1}{2}$ mark)

(c) Calculate: $x^2 - 3x$ where $x = -2$

10

($\frac{1}{2}$ mark)

(d) Solve $|2n - 3| < 6$ where n is an integer. Write all possible solutions.

$$\begin{array}{rclcl} -6 & < & 2n - 3 & < & 6 \\ -3 & < & 2n & < & 9 \\ -1.5 & < & n & < & 4.5 \end{array}$$

$$n \in \{-1, 0, 1, 2, 3, 4\}$$

or

- $n = -1,$
- $n = 0,$
- $n = 1,$
- $n = 2,$
- $n = 3,$
- $n = 4$

(1 mark)