



Mathematics Challenge 2015

by

Children's Well-wishers Network (CWN)

YEAR 9

Mark Scheme

We provide mark schemes of our CWN Mathematics Challenge 2015 examination papers to help parents.

Please note that for some problems there are more than one possible answer.

Some questions are open ended.

We strongly advise all children to practise the papers and think hard before looking at the answers provided.

Full answers and explanations will be provided on our feedback sessions.

In general, we expect units, directions, sensible answers and reasons in all questions.

Q1) The probability that it will rain tomorrow is one-third. The probability that it will rain day after tomorrow is one-sixth.

(a) What is the probability that it will rain at least one of the days, tomorrow or day after tomorrow?

$$1 - \frac{2}{3} \times \frac{5}{6}$$

$$= \frac{4}{9}$$

Or four-ninths

(1 mark)

(b) What is the probability that someone will hold an umbrella tomorrow?

We cannot judge
[as people hold umbrella for rain and for shade].

(1 mark)

(c) Siva rolls an ordinary fair four-sided dice 1200 times.

Work out an estimate for the number of times the dice will land on 1.

Any number close to 300

(1 mark)

(d) John has two fair 4-sided dice.

One dice is numbered 1, 2, 3 and 4. The other is numbered 2, 3, 4 and 5.

John throw both dice and multiply the scores to find the game score.

What is the probability that the game score is even?

You must show your working to explain your answer.

First die/second die	1	2	3	4
2	2	4	6	8
3	3	6	9	12
4	4	8	12	16
5	5	10	15	20

$$P(\text{game score is even}) = \frac{3}{4}$$

(1 mark)

Q2)

(a) Work out: 1006×994 using factorisation.

$$1000^2 - 6^2 = 1\,000\,000 - 36$$

$$= 999\,964$$

(1 mark)

(b) Factorise fully: $8x^3 - 1$

$$\equiv (2x - 1)(4x^2 + 2x + 1)$$

(1 mark)

Q3) The triangle A has sides of length 5, 6, 5. The triangle B has sides of length 5, 8, 5. What is the ratio of area A : area B? Show your working out.

$$Area = \sqrt{s(s-a)(s-b)(s-c)}$$

Ratio: 1 : 1

(2 marks)

Q4)

(a) How many significant figures does each of these numbers have? Write them in the brackets.

123 (3)

123,000 (3)

1.00 (3)

0.000123 (3)

(2 marks)

(b) When adding and subtracting numbers, the rules of significant figures require that the number of places after the decimal point in the answer is less than or equal to the number of decimal places in every term in the sum.

Find the sum of:

(i) 15600 and 172.49

15800 (1 mark)

(ii) 15600.00 and 172.49

15772.49 (1 mark)

(c) When multiplying and dividing numbers, the number of significant digits you use is simply the same number of significant figures as is the number with the fewest significant figures.

Find the product of: 15310 x 2.3

35000

(1 mark)

Q5) Soon after a bank robbery, five suspects were being questioned. Below is a summary of their statements. Police knew that each of them told the truth in one of the statements and lied in the other. From this information can you tell who committed the crime?

Bala said:

- "It wasn't Chandran."
- "It was Alan."

Das said:

- "It was Chandran."
- "It wasn't Alan."

Chandran said:

- "It was Bala."
- "It wasn't Ejaman."

Alan said:

- "It was Ejaman."
- "It wasn't Bala."

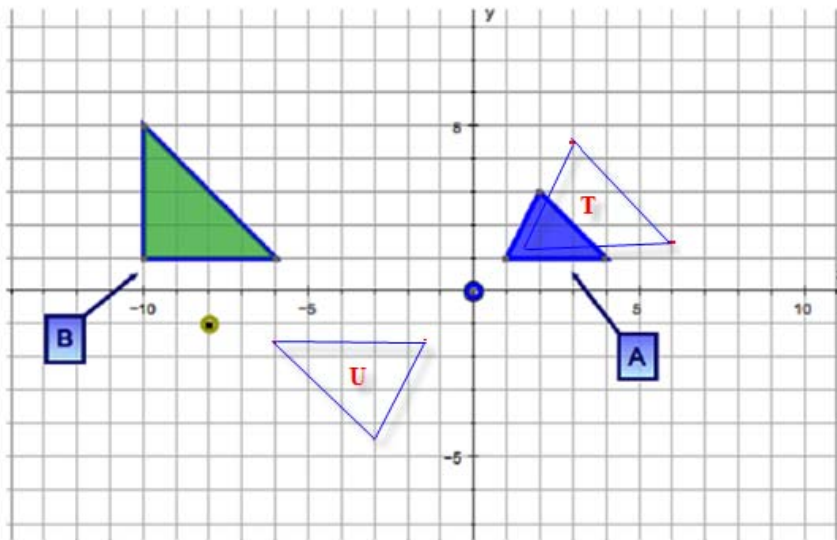
Ejaman said:

- "It was Das."
- "It was Alan."

Looking at Bala's statements, the first statement ("It wasn't Chandran.") cannot be false, otherwise this will contradict his second statement ("It was Alan."). Hence, his second statement must be false. This implies the culprit neither Chandran nor Alan. Looking at Ejaman's statements, as we know from Bala that the culprit wasn't Alan, Ejaman's second statement ("It was Alan.") must be false. This implies his first statement ("It was Das.") is true, and therefore the culprit was Das.

(2 marks)

Q6)



- (a) Enlarge shape A by scale factor 1.5 with centre (0, 0) as origin. Label the image T.

As shown on diagram above.

(1 mark)

- (b) Enlarge shape A by scale factor -1.5 with centre (0, 0) as origin. Label the image U.

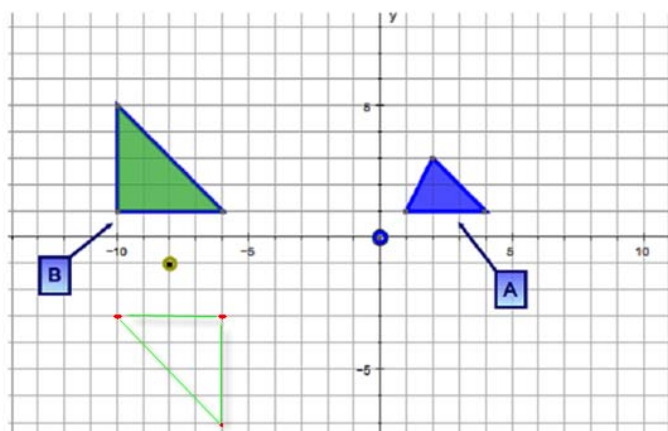
As shown on diagram above.

(1 mark)

- (c) Enlarge shape B by scale factor -1 with (-8, -1) as the centre of enlargement.

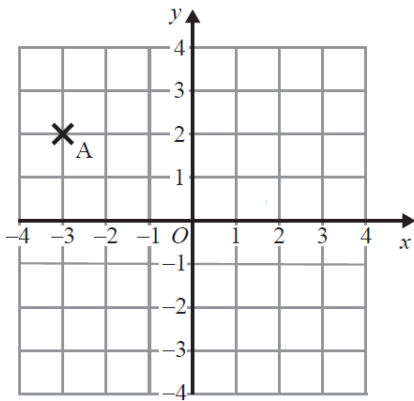
Write the co-ordinates of the vertices of resulting image.

$(-10, -3)$, $(-6, -3)$, $(-6, -7)$



(1 mark)

(d) Look at the graph below.



i. What is the co-ordinate of point A after rotating anticlockwise 90 degrees about the origin O?

$(-2, -3)$

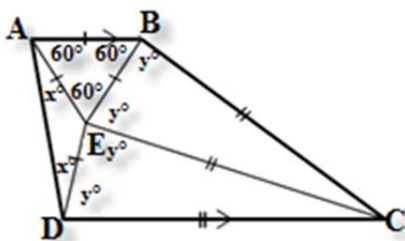
(1 mark)

ii. What is the co-ordinate of point A after reflection on $y = x$ line?

$(2, -3)$

(1 mark)

Q7) Using the geometrical indications for equal sides and parallel lines work out the size of angle $\angle ADE$.



$\angle ADE = 20^\circ$ as $\triangle CDE$ is congruent to $\triangle CBE$.

(application of congruency)

(1 mark)

Q8) The eight-digit number “xxxxyyyy”, where x and y are different digits, is divisible by 45.

What are the possible values of x?

$4(x+y)$ must be divisible by 9.
x can be 4 or 9.

Award $\frac{1}{2}$ mark for only “4” or only “9”

(1 mark)

Q9) L is a line with equation $y = 5x - 3$.

Write down the equation of a line:

(a) That is perpendicular and passing through origin.

$$y = -\frac{1}{5}x$$

(1 mark)

(b) That never intersects $y = 5x - 3$.

$$y = 5x + c \quad \text{where } c \neq -3$$

(1 mark)

Q10) The equation of the line of symmetry of a quadratic graph is $x = -2$. The maximum point of that graph is $(-2, 3)$.

(a) What is the equation of the graph?

Many possible answers of the form:

$$Y = k(x + 2)^2 + 3 \quad \text{where } k < 0$$

(1 mark)

(b) What is the equation of the above graph after translation to the right by 2 units?

Many possible answers of the form:

$$Y = k(x)^2 + 3 \quad \text{where } k < 0$$

(1 mark)

Q11) John wrote a book on humanity. He paged his book with number 1, 2, 3, and so on consequently. All in all he used 1152 digits to number all the pages of his book. What is the page number of the last page?

420

First nine (9) single digits: 1,2, ... 9
 Next ninety (90) double-digits : 10,11,... 99
 Next 321 triple digits 100, 101, .. 420

(1 mark)

Q12) Angles of a quadrilateral are in the ratio 1 : 2 : 3 : 14.

What is the difference between the largest angle and smallest angle?

234°

(1 mark)

Q13) A magician has two big boxes A and B with cats and mice (friendly together).

In box A, there are 5 cats and 3 mice.
 In box B, there are 4 cats and 5 mice.
 He now takes one animal from box A and put it in B. And then takes one animal from box B and put it in A.

What is the probability there will be 6 cats together in the same box?

$$\frac{3}{8} \times \frac{4}{10} = \frac{3}{20}$$

(1 mark)

Q14)

(a) Express $\frac{5}{37}$ as recurring decimal.

0.135135...

(1 mark)

(b) Work out:

$$0.31 - 0.3 \times 0.01 + 0.9999$$

1.3069

(1 mark)

(c) My calculator has a 11 digit display. The answer to a calculation is shown as 0.5555555555

i. Is this a rational number?
 Circle the correct answer.

Yes / No / **Cannot say**

(½ mark)

ii. Why?

Calculator truncates if a number is more than 11 digits. We do not know whether the number/numeric solution was 11 digits or more.

(½ mark)

Q15)

(a) Find the n^{th} term of these two sequences:

(i) 2, 11, 26, 47, 74, 107

Answer: $3n^2 - 1$

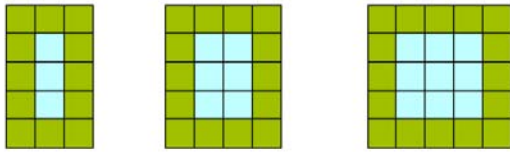
(1 mark)

(ii) -1, 5, 17, 35, 59, 89

Answer: $3n^2 - 3n - 1$

(1 mark)

(b) Here are 3 rectangular shapes.
How many squares will be on 10th shape?



Answer: **60** (1 mark)

Q16) The area of a circle is **numerically equal** to three times of its perimeter. Lengths are measured in centimetres (cm).
What is the measure of its radius?

6 cm (1 mark)

Q17) Student collected a vast amount of rainfall data over a year and calculated the mean of the average monthly rainfall in India. Will it give accurate picture of rainfall in India?

No. (½ mark)

Why?

There are so many unpredictable scientific factors plays into. No amount of statistical data can yield the accuracy of a chaotic nature.

(½ mark)

Q18) Given that $x : y = 2 : 3$
and $y : z = 2 : 3$

What is $x : z$?

4: 9 (1 mark)

Q19)

(a) Calculate the number which is halfway between 65×63 and 63×235

$$63 \times 150 = 9450$$

(1 mark)

(b) Write down the number halfway between 6.5×63 and 6.3×235

$$945$$

(1 mark)

Q20) The average age of a group of 10 students, in years, was 20.

The average age increased by 2 years when two new students joined the group.

What is the average age of the two new students who joined the group?

$$32$$

(1 mark)

Q21) Suppose that $8^m = 125$.

What is the value of 4^m ?

Show your working.

$$8^m = 125$$

$$\therefore 2^m = 5$$

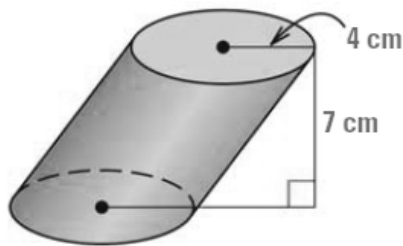
$$\therefore 4^m = 25$$

(2 marks)

Q22) Cavalieri's Principle says:

If two solids have the same height and the same cross-sectional area at every level, then they have the same volume.

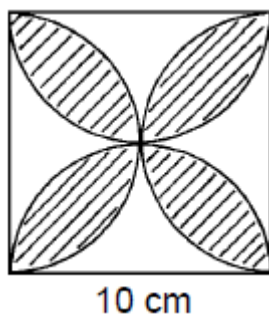
Using the above principle work out the volume of this oblique cylinder in multiples of π :



$$\begin{aligned} \text{Volume} &= \pi \times (4 \text{ cm})^2 \times 7 \text{ cm} \\ &= 112 \pi \text{ cm}^2 \end{aligned}$$

(2 marks)

Q23) Find the expression for the area shaded in terms of π .



$$\begin{aligned} \text{Area unshaded} + 2 \times \text{shaded} &= 4 \times \pi \times 5^2 \\ \text{Area unshaded} + \text{shaded} &= 10^2 \end{aligned}$$

$$\text{Shaded} = 100 \pi - 100$$

(2 marks)

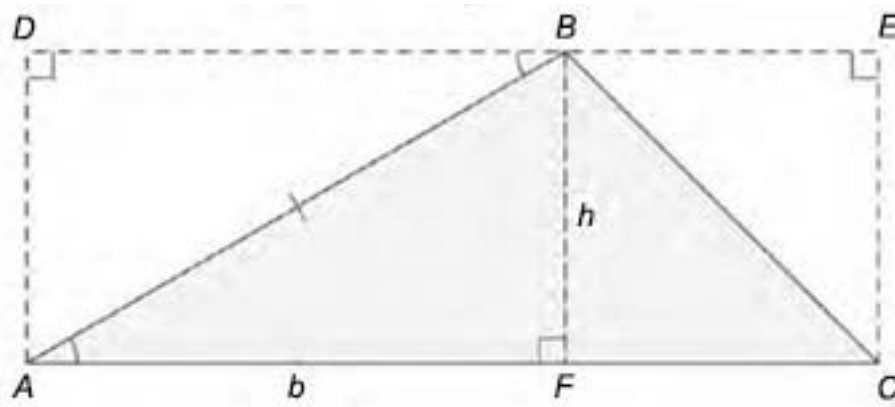
Q24) Seetha boils six eggs for her family's breakfast. She puts the eggs into the hot water pan one at a time, but waits one minute before putting the next egg in. If she boils each egg for exactly for three minutes, how long does the whole operation take from the moment she puts the first egg in to the moment she takes the sixth egg out?

8 min

(1 mark)

Q25)

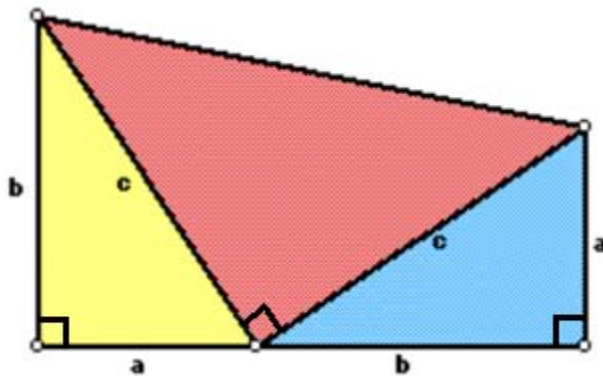
(a) Prove that area of the triangle is $\frac{1}{2}$ x base x height using the figure:



Area of rectangle = 2 x area of the triangle
 Area of rectangle = AC x AD
 \therefore Area of the triangle = $\frac{1}{2}$ x AC x AD
 Area of the triangle = $\frac{1}{2}$ x base x height

(1 mark)

(b) Using the following diagram, the above result and algebra prove the Pythagoras Theorem.



Area of the trapezium = $\frac{1}{2}$ x (a + b) x (a + b)
 = $\frac{1}{2}$ x (a² + 2ab + b²)
 = $\frac{1}{2}$ x (a² + b²) + ab
 Area of right-angled triangle = $\frac{1}{2}$ c²
 Area of blue triangle = $\frac{1}{2}$ ab
 Area of yellow triangle = $\frac{1}{2}$ ab
 Thus, $\frac{1}{2}$ ab + $\frac{1}{2}$ ab + $\frac{1}{2}$ c² = $\frac{1}{2}$ x (a² + b²) + ab
 Thus, c² = a² + b²

(1 mark)